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Mesons and nucleons from holographic QCD in a unified approach

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ABSTRACT: We investigate masses and coupling constants of mesons and nucleons within a hard wall model of holographic QCD in a unified approach. We first examine an appropriate form of fermionic solutions by restricting the mass coupling for the five dimensional bulk fermions and bosons. We then derive approximated *analytic* solutions for the nucleons and the corresponding masses in a small mass coupling region. In order to treat meson and nucleon properties on the same footing, we introduce the same infrared (IR) cut in such a way that the meson-nucleon coupling constants, i.e., $g_{\pi NN}$ and $g_{\rho NN}$ are uniquely determined. The first order approximation with respect to a dimensionless expansion parameter, which is valid in the small mass coupling region, explicitly shows difficulties to avoid the IR scale problem of the hard wall model. We discuss possible ways of circumventing these problems.

KEYWORDS: AdS-CFT Correspondence, Phenomenological Models

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1 Introduction

The AdS/CFT correspondence [1–3] that relates a strongly coupled large N_c gauge theory to a weakly coupled supergravity provides a novel approach to understand nonperturbative features of quantum chromodynamics (QCD) such as the quark confinement and spontaneous breakdown of chiral symmetry (SB χ S). Though no rigorous proof exists for such a correspondence in real QCD, this remarkable idea has triggered a great amount of theoretical works on possible mappings from nonperturbative QCD to 5 dimensional (5D) gravity, i.e. holographic dual of QCD. There are in general two different ways of modeling holographic dual of QCD (see, for example, a recent review [4]): One way is to construct 10 dimensional (10D) models based on string theory of D3/D7, D4/D6 or D4/D8 branes [5–9]. The other way is so-called a *bottom-up* approach to a holographic model of QCD, also known as AdS (Anti-de Sitter Space)/QCD [10–12] in which a 5D holographic dual is constructed from QCD based on the general wisdom of AdS/CFT, the 5D gauge coupling being identified by matching the two-point vector correlation functions. Despite the fact that this bottom-up approach is somewhat on an *ad hoc* basis, it reflects some of most important features of gauge/gravity dual. Moreover, it is rather successful in describing properties of mesons (see, for example, a recent review [4]).

On the other hand, QCD is not a conformal theory, in particular, in the low-energy region, so one should also incorporate this property in constructing an effective AdS/QCD model. Consequently, different models have been developed in this bottom-up approach. In refs. [10–12], the size of the extra dimension (also known as the compactification scale) was fixed at the point that corresponds approximately to the QCD scale parameter Λ_{QCD} , i.e., an infrared (IR) cutoff parameter was explicitly introduced. It is usually interpreted as the confinement scale that also breaks sharply the conformal invariance. These AdS/QCD models are called the *hard-wall model*. On the contrary, there is an alternative approach

called a *soft-wall model* in which the conformal invariance is broken smoothly by introducing the dilaton background field in the 5D AdS space [13–15].

While both approaches reproduce well meson properties such as masses, Regge trajectories, and so on, one serious problem arises, when it comes to fermions in the AdS/QCD. In order to describe the fermions, another IR cutoff was introduced in hard-wall models [16–21]. As shown in ref. [19], one has to introduce the IR cutoff for the nucleon with a different value from the mesonic case so that one may reproduce the excited nucleon spectra. In fact, ref. [19] used quite a small number for that. However, when one calculates the meson-nucleon coupling constants, an inconsistency arises [22]. In order to determine the coupling constants consistently, one must use the *same* IR cutoff. Otherwise, one cannot fully consider whole information on meson and nucleon wavefunctions. Thus, in the present work, we want to investigate the meson and baryon sectors on an equal footing with the same IR cutoff taken into account. To this end, we consider an anomalous dimension of baryon operator. We will also rederive the nucleon wavefunctions analytically in such a way that the analysis on the IR cutoff becomes easier.

The present work is organized as follows: In section 2, we briefly review a hard-wall model for mesons and nucleons. In section 3, we derive analytically the 5D energy eigenvalues and wavefunctions for the nucleons. In section 4, we discuss the meson-nucleon coupling constants. The results are presented and discussed in section 5. The last section is devoted to summary and conclusion.

2 Hard-wall model with holographic mesons and nucleons

We first briefly review a hard-wall model for mesons and for nucleons, developed in refs. [10, 11] and in ref. [19], respectively. The model has a geometry of 5D AdS

$$ds^2 = g_{MN} dx^M dx^N = \frac{1}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \quad (2.1)$$

where $\eta_{\mu\nu}$ stands for the 4D Minkowski metric: $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. The 5D AdS space is compactified by two different boundary conditions, i.e. the IR boundary at $z = z_m$ and the UV one at $z = \epsilon \rightarrow 0$. Thus, the model is defined in the range: $\epsilon \leq z \leq z_m$. Considering the global chiral symmetry $SU(2)_L \times SU(2)_R$ of QCD, we need to introduce 5D local gauge fields A_L and A_R of which the values at $z = 0$ play a role of external sources for $SU(2)_L$ and $SU(2)_R$ currents respectively. Since chiral symmetry is known to be broken to $SU(2)_V$ spontaneously as well as explicitly, we introduce a bi-fundamental field X with respect to the local gauge symmetry $SU(2)_L \times SU(2)_R$, in order to realize the spontaneous and explicit breakings of chiral symmetry in the AdS side. The current quark mass term $\bar{q}_L \hat{m} q_R$ with $\hat{m} = \text{diag}(m_u, m_d)$ breaks explicitly chiral symmetry, while its spontaneous breakdown is understood by the finite vacuum expectation value (VEV) of the quark condensate $\langle \bar{q}q \rangle$ that is regarded as an order parameter. Thus, considering these two, we can construct the bi-fundamental 5D bulk scalar field X in terms of the current quark mass m_q and the quark condensate σ

$$X_0(z) = \langle X \rangle = \frac{1}{2} (m_q z + \sigma z^3) \quad (2.2)$$

with isospin symmetry assumed. The current quark mass m_q is defined as $m_q = (m_u + m_d)/2$.

The 5D gauge action in AdS space with the scalar bulk field and the vector field can be expressed as

$$S_M = \int dz \int d^4x \sqrt{G} \text{Tr} \left[|DX|^2 + 3|X|^2 - \frac{1}{2g_5^2} (F_L^2 + F_R^2) \right], \quad (2.3)$$

where $\sqrt{G} = 1/z^5$, $DX = \partial X - iA_L X + iX A_R$, and $F_{L,R}^{MN} = \partial^M A_{L,R}^N - \partial^N A_{L,R}^M - i[A_{L,R}^M, A_{L,R}^N]$. The g_5 stands for the 5D gauge coupling and is fixed by matching the 5D vector correlation function to that from the operator product expansion (OPE): $g_5^2 = 12\pi^2/N_c$. The 5D mass of the bulk gauge field $A_{L,R}$ is determined by the relation $m_5^2 = (\Delta - p)(\Delta + p - 4)$ [2, 3] where Δ denotes the canonical dimension of the corresponding operator with spin p . The 5D mass of the bulk gauge field turns out to be $m_5^2 = 0$, which is natural because of gauge symmetry. Note that the vector and axial-vector gauge fields are defined as $V = (A_L + A_R)/\sqrt{2}$ and $A = (A_L - A_R)/\sqrt{2}$ that are coupled at the boundary to the vector current $J_V^{\alpha\mu} = \bar{q}\gamma^\mu t^a q$ and to the axial-vector current $J_A^{\alpha\mu} = \bar{q}\gamma^\mu \gamma_5 t^a q$ with $\text{tr}(t^a t^b) = \delta^{ab}$, respectively. The effective action describes the mesonic sector [10, 11] completely apart from exotic mesons [23].

Coming to the flavor-two ($N_F = 2$) baryonic sector, one needs to introduce a bulk Dirac field corresponding to the nucleon at the boundary [18, 21]. A specific hard-wall model for the nucleon was developed by ref. [19] and was applied to describe the neutron electric dipole moment [24] and holographic nuclear matter [25]. In this model, the nucleons are first introduced as a massless chiral isospin doublets (p_L, n_L) and (p_R, n_R) in such a way that the 't Hooft anomaly matching is satisfied. The spontaneous breakdown of chiral symmetry induces a chirally symmetric mass term for nucleons

$$\mathcal{L}_{\chi SB} \sim -M_N \begin{pmatrix} \bar{p}_L \\ \bar{n}_L \end{pmatrix} \Sigma (p_R, n_R) + \text{h.c.}, \quad (2.4)$$

where $\Sigma = \exp(2i\pi^a \tau^a / f_\pi)$ is the nonlinear pseudo-Goldstone boson field that transforms as $\Sigma \rightarrow U_L \Sigma U_R^\dagger$ under $SU(2)_L \times SU(2)_R$. The τ^a and f_π represent the $SU(2)$ Pauli matrices and the pion decay constant, respectively. Thus, we have to consider the following mass term in the AdS side

$$\mathcal{L}_I = -g \begin{pmatrix} \bar{p}_L \\ \bar{n}_L \end{pmatrix} X (p_R, n_R) + \text{h.c.}, \quad (2.5)$$

where g denotes the mass coupling (or Yukawa coupling) between X and nucleon fields, which is usually fitted by reproducing the nucleon mass $M_N = 940 \text{ MeV}$. In this regard, we can introduce two 5D Dirac spinors N_1 and N_2 of which the Kaluza-Klein (KK) modes should include the excitations of the massless chiral nucleons (p_L, n_L) and (p_R, n_R) , respectively. By this requirement, one can fix the IR boundary conditions for N_1 and N_2 at $z = z_m$.

Note that the 5D spinors $N_{1,2}$ do not have chirality. However, one can resolve this problem in such a way that the 4D chirality is encoded in the sign of the 5D Dirac mass

term. For a positive 5D mass, only the right-handed component of the 5D spinor remains near the UV boundary $z \rightarrow 0$, which plays the role of a source for the left-handed chiral operator in 4 dimension. It is vice versa for a negative 5D mass. The 5D mass for the $(d + 1)$ bulk dimensional spinor is determined by the AdS/CFT expression

$$(m_5)^2 = \left(\Delta - \frac{d}{2} \right)^2. \quad (2.6)$$

Since the nucleon consists of three valence quarks, the corresponding m_5 turns out to be $m_5 = 5/2$. However, since QCD does not have conformal symmetry in the low-energy regime, the 5D mass might acquire an anomalous dimension due to a 5D renormalization flow. Though it is not known how to derive it, we will introduce later some anomalous dimension of the m_5 to see its effects on the spectrum of the nucleon.

Considering all these facts, we are led to the 5D gauge action for the nucleons

$$\begin{aligned} S_N &= \int dz \int d^4x \sqrt{G} \text{Tr} [\mathcal{L}_K + \mathcal{L}_I], \\ \mathcal{L}_K &= i\bar{N}_1 \Gamma^M \nabla_M N_1 + i\bar{N}_2 \Gamma^M \nabla_M N_2 - \frac{5}{2} \bar{N}_1 N_2 + \frac{5}{2} \bar{N}_2 N_1 \\ \mathcal{L}_I &= -g \left[\bar{N}_1 X N_2 + \bar{N}_2 X^\dagger N_1 \right], \end{aligned} \quad (2.7)$$

where

$$\nabla_M = \partial_M + \frac{i}{4} \omega_M^{AB} \Gamma_{AB} - iA_M^L. \quad (2.8)$$

The non-vanishing components of the spin connection are $w_M^{5A} = -w_M^{A5} = \delta_M^A/z$ and $\Gamma_{AB} = \frac{1}{2i} [\Gamma^A, \Gamma^B]$ are the Lorentz generators for spinors. The Γ matrices in AdS are related to the 4D γ matrices as $\Gamma^M = (\gamma^\mu, -i\gamma_5)$.

3 Energy eigenvalues and eigenfunctions for nucleons

Since the mesonic sector is well studied in refs. [10, 11], we concentrate in this work on the nucleon properties from the AdS/QCD model. In order to find the mass spectrum of 4D nucleons, we have to solve the eigenvalue equations that arise from expanding N_1 and N_2 in terms of the KK eigenmodes. Decomposing the N_1 and N_2 fields into the following separable forms

$$\begin{aligned} N_1(x, z) &= f_{1L}(z) \psi_L(x) + f_{1R}(z) \psi_R(x), \\ N_2(x, z) &= f_{2L}(z) \psi_L(x) + f_{2R}(z) \psi_R(x), \end{aligned} \quad (3.1)$$

where $\psi_{L,R}$ denote the components of the 4D eigen-spinors $\psi = (\psi_L, \psi_R)^T$ with eigenvalues p , we obtain the coupled equations for $f_{1L,1R}$ and $f_{1R,2R}$

$$\begin{pmatrix} \partial_z - \frac{\Delta^+}{z} & -\phi(z) \\ -\phi(z) & \partial_z - \frac{\Delta^-}{z} \end{pmatrix} \begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix} = -p \begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix}, \quad (3.2)$$

$$\begin{pmatrix} \partial_z - \frac{\Delta^-}{z} & \phi(z) \\ \phi(z) & \partial_z - \frac{\Delta^+}{z} \end{pmatrix} \begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix} = p \begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix} \quad (3.3)$$

with the IR boundary conditions $f_{1L}(z_m) = f_{2L}(z_m) = 0$, where $\Delta^\pm = 2 \pm m_5$ and $\phi(z) = gX_0(z)z^{-1}$.

The left-handed eigenfunctions $f_{1L,2L}$ can be related to the right-handed ones $f_{1R,2R}$ for given parity states. Introducing the eigenvalues of the parity operator $P = \pm 1$, we can write the relations [19]

$$f_{2L} = -Pf_{1R}, \quad f_{2R} = Pf_{1L}. \quad (3.4)$$

Eq. (3.4) being used, eqs. (3.2) and (3.3) are reduced to

$$\left(\partial_z - \frac{\Delta^+}{z}\right) f_{1L} = -(p + P\phi)f_{1R}, \quad (3.5)$$

$$\left(\partial_z - \frac{\Delta^-}{z}\right) f_{1R} = (p - P\phi)f_{1L}. \quad (3.6)$$

These can be further decoupled as

$$f'' + A(z)f' + (p^2 - B(z))f = 0, \quad (3.7)$$

where f represents generically f_{1L} and f_{1R} . The f' and f'' are the first- and second-order derivatives with respect to z . The functions A and B are defined as

$$A(z) = -\partial_z \ln \left[(p \pm P\phi)z^{\Delta^- + \Delta^+} \right], \quad (3.8)$$

$$B(z) = \phi^2 - \frac{1}{z^2} (\Delta^- \Delta^+ + \Delta_\pm) \mp \frac{P\phi'\Delta^\pm}{z(p \pm P\phi)}. \quad (3.9)$$

The function f in eq. (3.7) satisfies, respectively the UV and the IR boundary conditions

$$f_L(\epsilon \equiv z_{UV} \rightarrow 0) = 0, \quad f_R(z_m \equiv z_{IR}) = 0, \quad (3.10)$$

which comes from the minimization of the action [16] and zeros of the right solutions f_R will be used to generate the mass spectrum of the nucleons.

Since eq. (3.7) can be put into an equation of the Sturm-Liouville type, we can write its solution in the form of¹

$$f(z) = Z(z) \exp \left\{ -\frac{1}{2} \int^z A(z) dz \right\} = Z(z) z^2 \sqrt{p \pm P\phi}. \quad (3.11)$$

Thus, we can consider eq. (3.7) as a simple quantum-mechanical 1D potential-well problem. Furthermore, introducing the following dimensionless variable and parameters

$$w = zz_m^{-1}, \quad \tilde{m}_q = \frac{g}{2} m_q z_m, \quad \tilde{\sigma} = \frac{g}{2} \sigma z_m^3, \quad \tilde{p} = pz_m, \quad (3.12)$$

we immediately obtain the dimensionless form of eq. (3.7)

$$Z''(w) + \left(\tilde{p}^2 - U(w) \right) Z(w) = 0, \quad (3.13)$$

¹We remark here that a similar method has been used in [26].

where the *effective* potential is defined as

$$U = U_0 + U_1, \tag{3.14}$$

$$U_0 = \frac{(m_5 \mp 1)m_5}{w^2} - \frac{P\tilde{\sigma}}{\tilde{p}}(2m_5 \pm 1) + \tilde{m}_q^2, \tag{3.15}$$

$$U_1 = \pm \frac{\tilde{\sigma}(\tilde{m}_q + \tilde{\sigma}w^2)}{\tilde{p}(\tilde{p} \pm P(\tilde{m}_q + \tilde{\sigma}w^2))} (2m_5 \pm 1) + \frac{3\tilde{\sigma}^2w^2}{(\tilde{p} \pm P(\tilde{m}_q + \tilde{\sigma}w^2))^2} + (2\tilde{m}_q + \tilde{\sigma}w^2)\tilde{\sigma}w^2. \tag{3.16}$$

Having examined the form of the solution given in eq. (3.11) and the potential in eq. (3.14)–(3.16), we find that the parameters \tilde{m}_q and $\tilde{\sigma}$ (and consequently g) are restricted by the singularity of the potential U_1 and by the structure of the corresponding even- or odd-parity solutions. Accordingly, the input parameters are restricted as follows:

$$|\tilde{m}_q + \tilde{\sigma}| = \frac{|g|}{2}(m_q + \sigma z_m^2)z_m < \tilde{p} \quad \text{or} \quad |g| < \frac{2p}{m_q + \sigma z_m^2} < g_{\text{crit}}. \tag{3.17}$$

Thus, we are able to restrict the mass coupling g in the present approach. Note that if one restricts the value of g by fitting it to the mass of the lowest state $p_1 = N(940)$, which is given as

$$\frac{2p_1}{m_q + \sigma z_m^2} \equiv g_{\text{crit}}, \tag{3.18}$$

then the singularity in the potential for all other states can be excluded. The meaning of the parameter-restriction condition can be easily understood. It measures the amount of the corrections to the energy states when chiral symmetry is broken, as obviously seen from eqs. (3.5)–(3.6).

We now examine two different limiting cases in the mass coupling: the limit of the *small mass coupling* $|g| \approx 0$ and the limit of the *strong mass coupling* $|g| \approx g_{\text{crit}}$. While in the limit of the small mass coupling the mass spectrum of the nucleons are almost the same as those in the restored phase of chiral symmetry, the changes in the spectrum turn out to be rather large in the opposite limit. A similar situation was already studied in the Nambu-Jona-Lasinio model [27] (see also relevant reviews [28, 29]).

The normalizable zero-mode solutions in the chirally symmetric phase is discussed in ref. [19]. After the spontaneous breakdown of chiral symmetry, one still has zero modes for fermions. To analyze this point, let us for the moment neglect the quark mass ($m_q = 0$) and look for the zero-mode solutions. One can easily find that the zero-mode equation has the form

$$Z''(w) - \left[\frac{n^2}{w^2} + \tilde{\sigma}^2w^4 \right] Z(w) = 0, \quad n^2 \equiv (m_5 \mp 1)m_5 \mp (2m_5 \pm 1) + 3. \tag{3.19}$$

This equation can be solved analytically and has the solution

$$Z(w) = w^{1/2} [C_1 I_{m_5 \mp 3/2}(\tilde{\sigma}w) + C_2 K_{m_5 \mp 3/2}(\tilde{\sigma}w)], \tag{3.20}$$

where $I_{m_5 \mp 3/2}$ and $K_{m_5 \mp 3/2}$ represent the modified Bessel functions. Taking into account eq. (3.11), we obtain the general zero-mode solution

$$f(w) = w^{5/2} \sqrt{\pm P \tilde{\sigma} w^2} [C_1 I_{m_5 \mp 3/2}(\tilde{\sigma} w) + C_2 K_{m_5 \mp 3/2}(\tilde{\sigma} w)], \quad (3.21)$$

which is proportional to the square root of the 5D mass coupling. One can save or kill the left- or the right-handed zero modes by appropriately choosing the sign of the mass coupling g .

If the conditions in eq. (3.17) are well satisfied, U_1 can be considered as a small perturbation of order $O(\tilde{\sigma}^2/\tilde{p}^2)$.² To the first order, i.e., when $U_1 = 0$, the problem can be solved analytically, so that

$$Z(w) = w^{1/2} (C J_{m_5 \mp 1/2}(a_P w) + D Y_{m_5 \mp 1/2}(a_P w)), \quad (3.22)$$

$$a_P^2 = \tilde{p}^2 + \frac{P \tilde{\sigma}}{\tilde{p}} (2m_5 \pm 1) - \tilde{m}_q^2, \quad (3.23)$$

where $J_{m_5 \mp 1/2}$ and $Y_{m_5 \mp 1/2}$ are the Bessel functions of the first and the second kinds, respectively. In order to get the finite solutions at the UV boundary, the coefficients $D_{L,R}$ must vanish.³ The energy levels of the states with a given parity correspond to the right-handed solutions (see eq. (3.10)). Consequently, one is led to the algebraic equations

$$\mu_n^2 = a_P^2(\tilde{p}_n), \quad (3.24)$$

where μ_n is the n^{th} zero of the Bessel function, i.e., $J_{m_5+1/2}(\mu_n) = 0$. Equation (3.23) has a prominent meaning. Since \tilde{p} gives in general the whole spectrum of the nucleon, one can immediately find that the mass of the first excited state with positive parity turns out to be smaller than that with negative parity. Thus, eq. (3.23) is consistent with the experimental data. That is, the ordering of the nucleon spectrum is analytically explained in this method.

The calculations of the next-order corrections are straightforward and can be done by using the expansion parameters, \tilde{m}_q/\tilde{p} and $\tilde{\sigma}/\tilde{p}$, and by expressing the potential U_1 in the form

$$U_1^{(R)} = (2\tilde{m}_q + \tilde{\sigma} w^2) \tilde{\sigma} w^2 - \frac{\tilde{\sigma}}{\tilde{p}^2} [(2m_5 - 1)(\tilde{m}_q + \tilde{\sigma} w^2) - 3\tilde{\sigma} w^2] + \frac{\tilde{\sigma}}{\tilde{p}^2} \sum_{n=1}^{\infty} [3(n+1)\tilde{\sigma} w^2 - (\tilde{m}_q + \tilde{\sigma} w^2)(2m_5 - 1)] \left(\frac{\tilde{m}_q + \tilde{\sigma} w^2}{\tilde{p}} \right)^n. \quad (3.25)$$

It will be shown that the corrections are rather small, so that this approximation works very well.

²See eq. (3.25). It is obvious that $\tilde{m}_q/\tilde{p} \ll \tilde{\sigma}/\tilde{p}$.

³The value of the UV boundary is taken to be zero in the present approach, i.e., $z_{UV} = 0$.

4 Meson-baryon couplings

Following ref. [11], one introduces the gauge-fixing terms

$$\mathcal{L}_{\text{gf}}^V = -\frac{1}{2\xi_V g_5^2 z} [\partial_\mu V^\mu - \xi_V h_1(V_5, z)]^2, \quad (4.1)$$

$$\mathcal{L}_{\text{gf}}^A = -\frac{1}{2\xi_A g_5^2 z} [\partial_\mu A^\mu - \xi_A (h_1(A_5, z) + h_2(X, z))]^2 \quad (4.2)$$

and can find an explicit form of the functions $h_{1,2}$ as done in refs. [19, 22]. In the unitary gauge $\xi_{A,V} \rightarrow \infty$ the fifth component of the vector field $V_5 = (L_5 + R_5)/2$ becomes infinitely heavy, so that it is decoupled from the theory. Analogously, the linear combination of the fifth component of the axial-vector field $A_z = (L_5 - R_5)/2$ and the bi-fundamental scalar field $X = v \exp\{iP\}$ components v and P becomes infinitely massive. Introducing the corresponding relation between $h_1(A_5, z)$ and $h_2(X, z)$ one can keep that linear combination massless. As a result the massless pions can be described. The corresponding equation for the pion mode function can be analytically integrated in the chiral limit and has the form

$$f_\pi = \frac{z^3}{N_0} \left[I_{2/3}(g_5 \sigma z^3/3) - \frac{I_{2/3}(g_5 \sigma z_m^3/3)}{I_{-2/3}(g_5 \sigma z_m^3/3)} I_{-2/3}(g_5 \sigma z^3/3) \right], \quad (4.3)$$

where $I_{\pm 2/3}$ are the modified Bessel functions. The normalization condition for the pion mode function is fixed by a canonical form of the kinetic term for the pion fields

$$\int_0^{z_m} dz \left[\frac{1}{2g_5^2 z} f_\pi^2 + \frac{z^3}{8X_0^2 g_5^4} \left(\partial_z \left(\frac{f_\pi}{z} \right) \right)^2 \right] = 1. \quad (4.4)$$

Once the pion fields are correctly identified, then the πNN coupling can be calculated as

$$g_{\pi N^{(n)} N^{(n)}} = \int_0^{z_m} dz \frac{1}{z^4} \left[f_\pi (f_{1L}^{(n)*} f_{1R}^{(n)} - f_{2L}^{(n)*} f_{2R}^{(n)}) - \frac{gz^2}{2X_0 g_5^2} \partial_z \left(\frac{f_\pi}{z} \right) (f_{1L}^{(n)*} f_{2R}^{(n)} - f_{2L}^{(n)*} f_{1R}^{(n)}) \right]. \quad (4.5)$$

Similarly, the ρNN coupling has the form of [22]

$$g_{\rho N^{(n)} N^{(n)}} = \int_0^{z_m} dz \frac{1}{z^4} [f_\rho + cz \partial_z f_\rho] \left[|f_{1L}^{(n)}|^2 + |f_{1R}^{(n)}|^2 \right], \quad (4.6)$$

where c is the constant of order in unity and the normalized ρ meson wave function has the following form

$$f_\rho = \frac{z J_1(m_\rho z)}{\left(\int_0^{z_m} dz z [J_1(m_\rho z)]^2 \right)^{1/2}}. \quad (4.7)$$

For completeness, we remind here the normalization condition for the mode functions of the baryons

$$\int_0^{z_m} \frac{dz}{z^4} \left(|f_{1L}^{(n)}|^2 + |f_{2L}^{(n)}|^2 \right) = 1 = \int_0^{z_m} \frac{dz}{z^4} \left(|f_{1R}^{(n)}|^2 + |f_{2R}^{(n)}|^2 \right). \quad (4.8)$$

5 Results and discussions

We now present the results of this work and discuss them. Most of input parameters of the model such as m_q , σ and z_m are quite well fitted in the mesonic sector [10]. Hence, we have only one free parameter g to reproduce the data in the baryonic sector. However, the IR cutoff z_m in the baryonic sector, which is often interpreted as a scale of the confinement, takes different values from those in the mesonic sector. Actually, ref. [19] performed two different fittings of these parameters. In the first fitting of ref. [19], the z_m and the σ were fixed in the mesonic sector, and the g is fitted to the nucleon mass. In the second fitting, the z_m and the g were taken respectively to be $(205 \text{ MeV})^{-1}$ and 14.4 such that the masses of the nucleon and the Roper resonance $N(1440)$ were reproduced. Since there is no reason for a nucleon to have the same scale of the confinement as that for a meson, this might be an acceptable argument as far as one treats mesons and baryons separately. However, there is one caveat. When it comes to some observables such as the meson-baryon coupling constants, we need to treat the mesons and baryons on the same footing and require inevitably a common z_m . Otherwise, we are not able to consider whole information on both mesons and baryons. Moreover, a model uncertainty brings on by the mass coupling g . Thus, in the present section, we will carry out the numerical analysis very carefully, keeping in mind all these facts.

We first take different values of the z_m from those in the mesonic sector and try to fit the data as was done in ref. [19]. In this case, σ is defined as $\sigma = 4\sqrt{2}(g_5 z_m^3)^{-1}$. Furthermore, we will examine two different limits of the mass coupling g : In the limit of the small mass coupling, there are three free parameters m_q , g and z_m . All other parameters can be related to z_m . On the other hand, in the limit of the strong mass coupling, the g can be fixed by eq. (3.18), which leaves only two free parameters. Obviously, the dependence on the current quark mass m_q must be tiny because of its smallness, so we can simply neglect it. In this case, we have only one free parameter.

The results of the calculations are listed in table 1. In the first part of the table, we present the results in the limit of the strong mass coupling. They are more or less the same as those obtained in ref. [19]. For comparison, we list the results for the small mass coupling in the middle part of table 1, and those of the leading-order approximation (see eq. (3.24)) in the last part, respectively. The mass coupling g is chosen to be 6 in both cases. While the spectrum of the nucleon seems to be qualitatively well reproduced, that of the ρ meson is fairly underestimated in comparison with the experimental data. In the case of the strong mass coupling, the situation becomes even worse. However, as dictated by eq. (3.23), the ordering of the nucleon-parity states are correctly reproduced for $0 < g < g_{\text{crit}}$.

The results listed in table 1 indicate that it is not possible to reproduce the spectra of the ρ meson and the nucleon at the same time.⁴ As an attempt to improve the above-presented results, we want to introduce an anomalous dimension of the 5D nucleon mass.

⁴Note that in the present work we do not aim at the fine-tuning of the parameters to reproduce the experimental data. The output data in baryonic sector is quite stable for changes in σ .

m_5	z_m^{-1}	$\sigma^{1/3}$	m_q	g	$(p, n)^+$	$N^+(1440)$	$N^-(1535)$	$\rho(776)$	$\rho(1475)$
The exact numerical results (the limit of the strong mass coupling)									
5/2	130.9*	126.4	0	15.9	940	1336.2	1366.5	314.8	722.6
5/2	129.7*	125.2	3*	15.7	940	1328.2	1357.5	311.8	715.7
5/2	126.0*	121.7	10*	15.4	940	1304.6	1331.9	303.0	695.5
The exact numerical results (the case of the small mass coupling)									
5/2	147.2*	142.1	0	6.0*	940	1440.3	1456.6	354.0	812.6
5/2	147.0*	141.9	3*	6.0*	940	1439.3	1455.6	353.5	811.5
5/2	146.3*	141.3	10*	6.0*	940	1434.6	1451.0	351.8	807.6
The leading order approximation (the case of the small mass coupling)									
5/2	147.2*	142.1	0	6.0*	919	1428.4	1445.1	354.0	812.6
5/2	147.0*	141.9	3*	6.0*	918	1426.5	1443.1	353.5	811.5
5/2	146.3*	141.3	10*	6.0*	914	1415.0	1436.6	351.8	807.6

Table 1. The results of the spectra of the nucleon and the ρ meson. In the limit of the small mass coupling, there are three free parameters m_q , g and z_m , while in the limit of the strong mass coupling, the g is fixed near its critical value (see eq. (3.18)). All dimensional quantities are expressed in units of MeV. The asterisks indicate input parameters. The parameter σ is defined as $\sigma = 4\sqrt{2}(g_5 z_m^3)^{-1}$. The results of the leading-order approximation are yielded according to eq. (3.24). The 5D nucleon mass m_5 is given by the AdS/CFT, eq. (2.6).

m_5	z_m^{-1}	$\sigma^{1/3}$	m_q	g	$(p, n)^+$	$N^+(1440)$	$N^-(1535)$	$\rho(776)$	$\rho(1475)$
Model A									
0	323*	327*	2.29*	-1.5	1009	2029	2036	776	1783
1	323*	327*	2.29*	1	1449	2495	2498	776	1783
2	323*	327*	2.29*	2.1	1853	2936	2949	776	1783
5/2	323*	327*	2.29*	2.5	2050	3150	3168	776	1783
Model B									
0	346*	308*	2.30*	-2.4	1081	2031	2040	832	1910
1	346*	308*	2.30*	1.3	1553	2494	2499	832	1910
2	346*	308*	2.30*	3.1	1985	2937	2952	832	1910
5/2	346*	308*	2.30*	3.6	2196	3151	3172	832	1910

Table 2. The results of the spectra of the nucleon and the ρ meson with the 5D mass m_5 varied in the range of $0 \leq m_5 \leq 5/2$. The mass coupling g is fitted to the spectrum of the nucleon. All other parameters are taken from ref. [10]. All dimensional quantities are presented in units of MeV. The asterisks indicate input parameters. Two different sets of input parameters are used as in ref. [10].

Note that the 5D mass of the bulk vector field does not acquire any anomalous dimension because of the gauge symmetry.

Table 2 lists the results of calculations for different values of the 5D mass m_5 , whereas the z_m , the σ , and m_q are fitted to the mesonic sector. Note that here the nucleon mass is not used as an input. Varying the value of g , we try to fit the spectrum of the nucleon. We present the results from two different parameter sets called model A and model B. In

z_m^{-1}	$\sigma^{1/3}$	g	$(p, n)^+$ (939)	N^+ (1440)	N^- (1535)	ρ (776)	ρ (1475)	$g_{\pi NN}$ (13.1)	$g_{\rho NN}$ (2.4)
285*	256*	-2.0*	890	1791	1797	685	1573	1.65	1.39
285*	237*	-2.0*	890	1790	1796	685	1573	1.76	1.39
285*	256*	-8.0*	930	1826	1856	685	1573	4.89	1.34
285*	237*	-8.0*	920	1817	1843	685	1573	5.12	1.35
285*	227*	-9.6*	930	1826	1856	685	1573	6.12	1.34
280*	252*	-2.0*	874	1760	1765	673	1546	1.65	1.39
280*	233*	-2.0*	874	1759	1764	673	1546	1.76	1.39

Table 3. The results of the spectra of the nucleon and the ρ meson and the πNN and ρNN coupling constants. The parameters z_m , σ , and g are found by the global fitting procedure. The anomalous dimension of the 5D nucleon mass is chosen in such a way that the 5D mass vanishes. All other definitions are the same as in tables 1 and 2.

this analysis, we take the values of the z_m and σ from ref. [10]. Note that the ρ meson mass is used as an input in model A, while model B corresponds to the global fitting done in ref. [10]. The 5D nucleon mass is varied in the range of $0 \leq m_5 \leq 5/2$, its anomalous dimension being considered as mentioned before. As shown in table 2, the best result is obtained with $m_5 = 0$. Though the absolute values of the nucleons turn out to be overestimated in contrast to the previous analysis presented in table 1, the ground-state masses of the nucleon and the ρ meson are qualitatively well reproduced within 30%.

We are now in a position to include meson-baryon coupling constants in the present numerical analysis. We will consider here the πNN and the ρNN coupling constants in addition to the ρ meson and the nucleon spectra. One has to keep in mind that in order to calculate the meson-baryon coupling constants it is essential to use the same z_m for the mesonic and baryonic sectors. Otherwise, it is not possible to keep whole information on the wavefunctions. Thus, it is of utmost importance to compute all observables with the same set of parameters. We perform a global fitting procedure to obtain the results listed in table 3. Note that we consider here the chiral limit ($m_q = 0$), since its effects on the results are rather tiny.⁵ We assume also that the 5D nucleon mass acquires a large anomalous dimension so that it may vanish, i.e., $m_5 = 0$. The best fit is obtained with the parameters fitted as follows: $z_m = (285 \text{ MeV})^{-1}$, $\sigma = (227 \text{ MeV})^3$, and $g = -9.6$. The masses of the ground-state nucleon and the ρ meson are in good agreement with the experimental data. Moreover, those of the excited states are qualitatively well reproduced within 10 – 20%. However, the coupling constants are in general about 50% underestimated. We mention that in ref. [22] the dependence of the meson-baryon coupling constants on the z_m was investigated without considering hadron spectra but the results for the coupling constants are more or less in the same level as in the present work.

⁵Note that in the chiral limit, the nucleon mass is different from experiments, $M_n \simeq 939 \text{ MeV}$. For instance, $M_n \simeq 882 \text{ MeV}$ in the chiral limit [30].

6 Summary

We have investigated the mesons and the nucleons in a unified approach, based on a hard-wall model of AdS/QCD [10, 19]. We first have decoupled the equations of motion for the nucleons and casted them into the Sturm-Liouville type such that the problem for the nucleons is reduced to a simple one dimensional quantum-mechanical potential-well problem. In order to study the nucleon spectrum, we developed an approximated method in which the effective potential can be expanded. The method of this approximation was shown to work very well. In particular, the correct ordering of the nucleon parity states was *analytically* shown in this method.

We then have carried out various numerical analyses, varying the model parameters such as the IR cutoff z_m , the quark condensate σ , and the mass coupling (or Yukawa coupling) g . In order to improve the results of the nucleon and the ρ meson spectra on an equal footing, we have introduced an anomalous dimension of the 5D nucleon mass. We found that the zero 5D nucleon mass, $\Delta = 2$, produces the best results.

We finally have included the πNN and ρNN coupling constants. Upon calculating the coupling constants, it is essential to use the same z_m for mesons and nucleons, so that whole information about the wavefunctions are not lost in the course of the calculation. We have performed the global fitting procedure in which we obtained the best fit with the values of the parameters: $z_m = (285\text{MeV})^{-1}$, $\sigma = (227\text{MeV})^3$, and $g = -9.6$. The mass spectra of the nucleon and the ρ meson are in relatively good agreement with the experimental data within 10 – 20%, whereas the πNN and ρNN coupling constants are underestimated by about 50%.

In order to improve the present results, one might consider higher dimensional operators [31], or a finite UV cutoff [32].

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